CSCE 411
Max Flow and Bipartite Matching

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1. In the *stable roommates* problem, you are given as input a set of $2n$ students (where n is some integer) and a preference list for each student ranking all $2n-1$ of the other students. The goal is to find a way to arrange the students into roommates such there is no unstable pair. As in the stable matching problem, we define an unstable pair as a pair that prefer each other over their current partners.

   Our proof that Gale-Shapley always finds a stable matching proves as a side effect that the stable matching problem always has a solution. Prove that the stable roommates problem does *not* always have a valid solution.

2. Managers at Bureaucracy Inc. are concerned that the company isn't getting anything done. They've decided that the natural response is to form a collection of committees aimed at addressing this issue. You are given:

   - The number of workers x, and the number of committees y.
   - An array of positive integers $A[1, \ldots, x]$ where $A[i]$ is the maximum number of committees that the i'th worker can handle serving on.
   - A 2D array of booleans $B[1, \ldots, x][1, \ldots, y]$ where $B[i][j]$ is true if the i'th worker can serve on the j'th committee.
   - An array of positive integers $C[1, \ldots, y]$ where $C[i]$ is the number of members that the i'th committee needs

   Your goal is to determine whether there exists an assignment of workers to committees such that each worker is assigned only to committees that they are able to serve on, each committee has as many members as it needs, and no worker is serving on more committees than they can handle. Give an efficient algorithm for this problem.

3. Give a polynomial time algorithm to determine if an instance of the stable matching problem has a *unique* solution (i.e. if only one possible matching is stable).

4. Prove that a graph is bipartite if and only if it has no cycles of odd-length.