

CSCE 411

Graph Algorithms

1. Ryan is heading to a birthday party, and along the way he needs to pick up a birthday cake and a present. Of course, as a computer scientist, he wants to find the fastest route to get those two things and then get to the party. You are given:
 - An undirected weighted graph G where the vertices of G are locations and the weight of an edge $\{u, v\}$ represents the number of minutes it takes to get between u and v .
 - A vertex $s \in V(G)$ representing Ryan's starting location
 - A vertex $t \in V(G)$ representing the location of the party
 - A set $C \subseteq V(G)$ representing the locations that sell cake.
 - A set $P \subseteq V(G)$ representing the locations that sell acceptable presents.

Give an efficient algorithm to find the fastest route that Ryan can take to acquire a present and a cake and then get to the party (note that the present and cake can be acquired in either order). Analyze the runtime of your algorithm.

2. You want to get together with a childhood friend who lives in a different state. In order to meet up as soon as possible, you decide to meet at an intermediate city. How can you choose an intermediate city that lets the two of you meet as soon as possible, assuming that you both leave right now?

You are given a directed weighted graph G where the vertices are cities and the weight of an edge (u, v) is the time to get from u to v . You are also given a city x which is your location and a city y which is your friend's. Give an efficient algorithm to find the intermediate city that will allow the two of you to meet as soon as possible. Analyze the runtime of your algorithm.

3. We say that a pair of vertices u and v in a directed graph G are incomparable if G has neither a $u \rightarrow v$ path nor a $v \rightarrow u$ path.
 - (a) Give an example of a 4-vertex directed graph with no incomparable pair. Give an example of a 4-vertex directed graph with an incomparable pair where each vertex in the pair has at least one incoming edge and at least one outgoing edge.
 - (b) Give an $O(m + n)$ time algorithm to determine if a directed acyclic graph has an incomparable pair.
 - (c) Give an $O(m + n)$ time algorithm to determine if an arbitrary directed graph has an incomparable pair.