

CSCE 411

Greedy Algorithms and Graphs Practice

1. For a positive integer k and a graph $G = (V, E)$, a k -coloring of G is a function $c : V \rightarrow \{1, \dots, k\}$ such that $\forall \{u, v\} \in E, c(u) \neq c(v)$. That is, a k -coloring assigns each vertex one of k colors such that no two adjacent vertices receive the same color. We say that G is k -colorable if it has a k -coloring.

Let d be a positive integer, and let G be a graph with maximum degree d . Prove that G is $(d + 1)$ -colorable [Hint: Give a greedy algorithm to color G].

2. For a graph $G = (V, E)$ we define the *complement* of G , denoted $\overline{G}$ as

$$\overline{G} := (V, \{\{x, y\} \subseteq V : x \neq y \wedge \{x, y\} \notin E\}).$$

That is, $\overline{G}$ is a graph on the same vertex set with all of the edges which are not in G . Prove that if G is disconnected then $\overline{G}$ is connected.

3. Prove that for every even integer n and every non-negative integer $k < n$, there exists a k -regular graph on n vertices.
4. Recall the job scheduling problem: We are given a set of n jobs J which our machine needs to run. Each job j has two attributes:
 - $j.time$ is the time that j takes our machine to complete (measured e.g. in minutes)
 - $j.deadline$ is the time by which we are supposed to complete j (measured in minutes since the time when we start running jobs).

We need to select an order for the jobs to run in. In the notes we saw a greedy algorithm for this problem which minimizes the maximum lateness. Suppose that instead we want to maximize the number of jobs completed on-time (i.e. at or before their deadline). Give an efficient algorithm to find an optimal ordering for this objective. Your algorithm should run in $O(n^2)$ time or better.

5. Let G be a graph with n vertices and at least $n^2/4$ edges. Prove that G has a subgraph G' with minimum degree at least $n/4$ (i.e. $\min_{v \in V(G')} \deg_{G'}(v) \geq n/4$).