CSCE 411
Asymptotic Analysis Practice

---

1. Prove that $\frac{\sqrt{n}-\sqrt[3]{n}}{2} \in \Omega(\sqrt{n})$ using the formal definition of big-Ω.

2. Analyze the runtime of this algorithm and give a big-Θ bound (in terms of n).

```
ShiftySort(A[1, ..., n]):
    For i from 1 to n − 1:
        For j from n − i to n − 1:
            If A[j] > A[j + 1]:
                Swap A[j] with A[j + 1]
            Else:
                Break
```

3. Suppose that f, g, f', and g' are all functions from $\mathbb{Z}^+$ to $\mathbb{Z}^+$ with $f'(n) \in O(f(n))$ and $g'(n) \in O(g(n))$. For each of the following statements either prove it, or disprove it by giving a counterexample:

   (a) $\max(f'(n), g'(n)) \in O(\max(f(n), g(n)))$

   (b) $f'(n)^{g'(n)} \in O(f(n)^{g(n)})$

   (c) $f'(n)/g'(n) \in O(f(n)/g(n))$

   (d) $f'(g'(n)) \in O(f(g(n))$

4. Let $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ and $g : \mathbb{Z}^+ \to \mathbb{Z}^+$ be two functions. Prove that if $f(n) = O(g(n))$ and $g(n) = O(f(n))$ then $f(n) = \Theta(g(n))$

5. Analyze the runtime of this algorithm and give a big-Θ bound in terms of the input length n. Assume that x and y are given in binary (How long do the arithmetic operations inside the loop take?)

```
RPM(x, y)
    r ← 0
    a ← x
    b ← y
    While b > 0:
        If b mod 2 = 1 :
            r ← r + a
        a ← 2a
        b ← ⌊b/2⌋
    Return r
```